Relative collision safety in cars

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Abstract

We propose a new model for relative collision safety in cars. Our present research are restricted to head-on crashes between two cars. When two cars crash they are exposed to the same force, but the damage severity is different depending on various factors such as car mass, change of speed and design of the car. The underlying mathematical model is partially founded on birth processes. Police and hospital data are used to explore the relationships in our model.

1 Introduction

In this paper we investigate the possibility of extracting a relative collision safety risk, α, between different car models. It is an interesting problem to study because almost everyone drives a car in their every-day-life and most people care about their own, and hopefully others, safety in case of a collision. One question of interest might be how safe one car model, for example Volvo, is compared to another model, for example Saab, and this is what we call the relative collision safety risk. This work is inspired by Tingwall and others work at the insurance company Folksam , see Hägg et al (1992). We study similar questions, but our model is new. The aim of this study is to examine this car model dependent risk, α. The analysis is restricted to head-on crashes between two cars in which at least one of the two drivers involved was injured. Data from Statistics Sweden (SCB) containing traffic accidents reported to the police during the years 1992-1993 are used to explore the relationships in our model. Known quantities from this data set are car model, car mass and the injury of the driver.

The injuries are classified according to:

0: The driver is unhurt.

1: The driver is mildly hurt.

2: The driver is seriously hurt.

3: The driver is dead within 30 days.

The injury classification according to the police are completed with the hospital injury register to improve the classification.

2 Physical Background

When two cars crash head-on into each other they are exposed to the same force, but the damage severity is different. The difference is depending on various factors such as the masses of the two cars involved, the change of speed that the vehichle undergoes and the design of the car - a measure connected to car model. In Evans (1994a,b) relationships between driver risk and car mass is studied. This study is an effort to absolute how much of the injury risk that depends on car model.

I call the two cars involved in a crash car number one and car number two. The enumeration depends only on where in my database they appear. Let

m_1 = the mass of car number one
m_2 = the mass of car number two.
v_{11} = the speed of car number one before the crash.
v_{21} = the speed of car number two before the crash.
v_{12} = the speed of car number one after the crash.
v_{22} = the speed of car number two after the crash.
Δv_1 = the change of speed that car number one undergoes in the crash
Δv_2 = the change of speed that car number two undergoes in the crash

Newtonian mechanics helps us to tie the two cars in the crash together. The law of conservation of linear momentum states that linear momentum before crash equals linear momentum after crash. I.e.

$$m_1 v_{11} + m_2 v_{21} = m_1 v_{12} + m_2 v_{22}$$

$$\Delta v_2 = \frac{m_1}{m_2} \Delta v_1$$

3 Notations

Let θ be a nuisance parameter connected to the change of speed that the vehicle undergoes in the crash. Let

θ_1 be proportional to Δv_1 and
θ_2 be proportional to Δv_2 i.e. proportional to $\frac{m_1}{m_2} \Delta v_1$

Further I let α be my so called design parameter. It is my parameter of interest and it is connected to car model. Each car model corresponds to a particular α. We have grouped versions of the same car model together, i.e there is one α for Volvo, one for Saab etc. This car model dependent risk, α should be interpretended in the following way: the smaller the alpha, the more secure is the car model.

Let t be a measure on how much violence the driver is exposed to in the crash, i.e. t depends on car mass, change of speed and car model: $t = t(m, \Delta v, \alpha)$.

As usual we let P(.) stand for probability:

$P_0(t)$ = P(the driver will end up in injury class 0 exposed to violence t)
$P_1(t)$ = P(the driver will end up in injury class 1 exposed to violence t)
$P_2(t)$ = P(the driver will end up in injury class 2 exposed to violence t)
$P_3(t)$ = P(the driver will end up in injury class 3 exposed to violence t)

4 Model formulation

Trying to find a suitable mathematical model the following criteria were put: $P_0(t)$ should be cloose to one when t is small and then decrease to zero when t increases. This means that if there is a gentle crash the probability should be close to one that the driver will be unhurt. On the opposite $P_3(t)$ should be cloose to zero when t is small and then increase to one when t increases. $P_1(t)$ and $P_2(t)$ should increase from zero to a maximum and then decrease back to zero again. Of course $P_1(t)$ and $P_2(t)$ vary only between zero and one. $P_1(t)$ should be shifted towards the left compared to $P_2(t)$.

All our demands are fulfilled if we model this as a pure birth-process where we let the states correspond to the injury classes. This means that we have a birth process where the states have the following interpretation:

State	Interpretation
0	The driver has ended up in injury class 0
1	The driver has ended up in injury class 1
2	The driver has ended up in injury class 2
3	The driver has ended up in injury class 3

Introducing a birth process forces us to introduce further nuisance parameters, namely the birth rates: λ_0, λ_1, λ_2, λ_3.

5 Calculations

Introducing a birth model we have now the possibility to calculate the transition probabilities P_0, ..., P_3. Kolmogorovs Forward Equations for a pure birth process states that:

$$P_{ii}(t) = e^{-\lambda_i t} \qquad i \geq 0$$

$$P_{ij}(t) = \lambda_{j-1} e^{-\lambda_j t} \int_0^t e^{\lambda_j s} P_{i,j-1}(s) ds \qquad j \geq i+1$$

In our model we always start at state 0 (we assume that the driver is unhurt before the crash). This means that $i = 0$ all the time and the index i is not necessary. Kolmogorovs Equations simplifies to :

$$P_0(t) = e^{-\lambda_0 t}$$

$$P_j(t) = \lambda_{j-1} e^{-\lambda_j t} \int_0^t e^{\lambda_j s} P_{j-1}(s) ds \qquad j \geq 1$$

While state number three is the highest state in our model, the only possible value of λ_3 is zero. Solving the equations (with $\lambda_3 = 0$) gives us:

$$P_0(t) = e^{-\lambda_0 t}$$

$$P_1(t) = \frac{\lambda_0}{\lambda_1 - \lambda_0} e^{-\lambda_0 t} + \frac{\lambda_0}{\lambda_0 - \lambda_1} e^{-\lambda_1 t}$$

$$P_2(t) = \frac{\lambda_0 \lambda_1}{(\lambda_1 - \lambda_0)(\lambda_2 - \lambda_0)} e^{-\lambda_0 t} + \frac{\lambda_0 \lambda_1}{(\lambda_0 - \lambda_1)(\lambda_2 - \lambda_1)} e^{-\lambda_1 t} + \frac{\lambda_0 \lambda_1}{(\lambda_0 - \lambda_2)(\lambda_1 - \lambda_2)} e^{-\lambda_2 t}$$

$$P_3(t) = 1 - \frac{\lambda_1 \lambda_2}{(\lambda_2 - \lambda_0)(\lambda_1 - \lambda_0)} e^{-\lambda_0 t} - \frac{\lambda_0 \lambda_2}{(\lambda_0 - \lambda_1)(\lambda_2 - \lambda_1)} e^{-\lambda_1 t} - \frac{\lambda_0 \lambda_1}{(\lambda_0 - \lambda_2)(\lambda_1 - \lambda_2)} e^{-\lambda_2 t}$$

6 Estimation

The vector of parameters that we want to estimate is $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, ..., \alpha_m)$, where m eqals the number of car models in the study. To estimate α we have to estimate several nuisance parameters, namely $\boldsymbol{\theta_1} = (\theta_{11}, \theta_{12}, ..., \theta_{1n})$, $\boldsymbol{\theta_2} = (\theta_{21}, \theta_{22}, ..., \theta_{2n})$, where n eqals the number of crashes in the study, and the birth rates $\lambda_0, \lambda_1, \lambda_2$. As described above, λ_3 must equal to zero. Further we standardize λ_0 to one. Using that $\theta_{2j} = \frac{m_{1j}}{m_{2j}}\theta_{1j}$ leaves us with $k+n+2$ parameters to estimate.

Let us study one example of how we describe the probabilities after one crash, for example crash number 109 in our data set. Letting l stand for the number of the crash in our data set, $l = 109$ in this example ($l = 1, ..., n$). In every crash there are two cars involved: car number one and car number two. We let k be the index connected to the number of the car, i.e. $k = 1, 2$.

	Car number one $k = 1$	Car number two $k = 2$
Crash number	$l = 109$	$l = 109$
Car model	Vw	Volvo
Relative risk	α_{Vw}	α_{Volvo}
Injury class	j = 1	j = 0
Car mass	$m_{1,109} = 1010$ kg	$m_{2,109} = 1120$ kg
Violence	$t_1 = \theta_{1,109}\alpha_{Vw}$	$t_2 = \theta_{2,109}\alpha_{Volvo} = \frac{m_{1,109}}{m_{2,109}}\theta_{1,109}\alpha_{Volvo}$
Probabilities from Kolmogorovs Eq.	$P_{j_{k,l}} = P_{1_{1,109}} = \frac{\lambda_0}{\lambda_1-\lambda_0}e^{-\lambda_0\theta_{1,109}\alpha_{Vw}} + \frac{\lambda_0}{\lambda_0-\lambda_1}e^{-\lambda_1\theta_{1,109}\alpha_{Vw}}$	$P_{j_{k,l}} = P_{0_{2,109}} = e^{-\lambda_0\frac{m_{1,109}}{m_{2,109}}\theta_{1,109}\alpha_{Volvo}}$

In order to estimate the parameters we use the Maximum Likelihood method. If we just studied crash number 109 the Likelihood would be:

$$L = P_{1_{1,109}}P_{0_{2,109}} = (\frac{\lambda_0}{\lambda_1 - \lambda_0}e^{-\lambda_0\theta_{1,109}\alpha_{Vw}} + \frac{\lambda_0}{\lambda_0 - \lambda_1}e^{-\lambda_1\theta_{1,109}\alpha_{Vw}})e^{-\lambda_0\frac{m_{1,109}}{m_{2,109}}\theta_{1,109}\alpha_{Volvo}}$$

In the full problem the Likelihood contains 2n factors and knowing which of the probabilities $P_0, ..., P_3$ to choose in every crash we are now able to calculate the Likelihood, L.

$$L = \prod_{k,l} P_{j_{k,l}}, k = 1, 2; l = 1, ..., n$$

The log-Likelihood is maximized in an estimation procedure, where the three different parameter vectors: the $\boldsymbol{\alpha}$, the $\boldsymbol{\theta}$ and the $\boldsymbol{\lambda}$ are estimated iteratively in three different steps. The estimation procedure works fine and since the project is still developping the results will be given in a forthcoming licenciate thesis.

7 References

Hägg A, Kamren B, v Koch M, Kullgren A, Lie A, Malmstedt B, Nygren Å , Tingwall C. (1992), "Folksam Car Model Safety Rating 1991-1992", FOLKSAM research.

Evans, L. (1994 a), "Driver Injury and Fatality Risk in Two-Car Crashes Versus Mass Ratio Inferred Using Newtonian Mechanics", Accident Analysis and Prevention, 26.

Evans, L. (1994 b), "Small Cars, Big Cars: What Is the Safety Difference?", Chance, Vol.7, No3.