Elementary units of exposure

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Outline of presentation

- Foundations of the concepts of exposure and risk in probability theory
- Commonly used measures of exposure and their weaknesses
- Developing event-based measures of the number of opportunities for accidents based on easily available summary measures of exposure
- The shape of the relationship between exposure and the number of accidents
- Implications for the applicability of commonly accepted probability models

The Poisson probability law

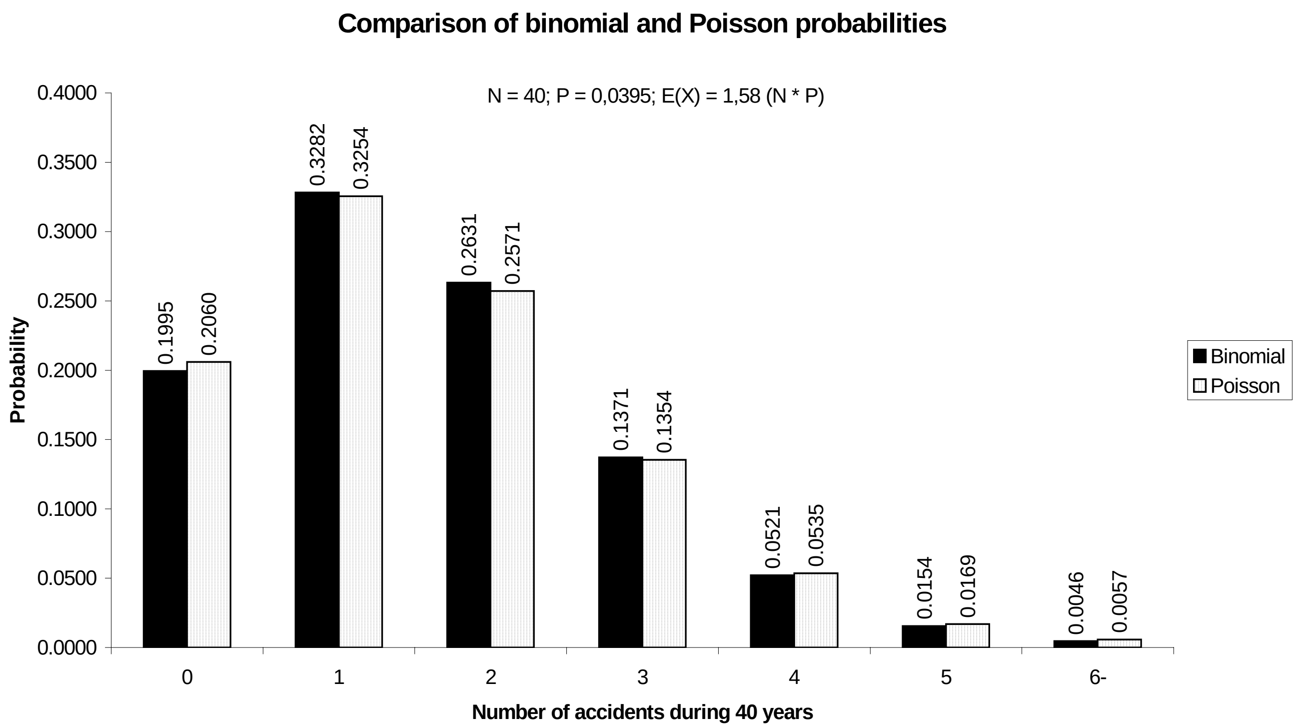
- Poisson derived his law as a limiting case of a set of binomial trials (Bernoulli trials)
- Binomial trials:
  - There are two outcomes of each trial (success or failure)
  - The probability of these outcomes is the same at each trial
  - The outcome of any trial is independent of the outcome of other trials
- The binomial limit theorem:
  - When the number of trials, N, goes toward infinity, and
  - The probability of failure at a given trial, p, goes toward zero,
  - The distribution of the number of failures in N trials is approximated by the Poisson distribution
- Von Bortkiewicz was the first researcher to describe accident occurrence as the outcome of Poisson trials

Application and interpretation

$$\lambda = N \cdot p$$

$$\text{Expected number of accidents } (\lambda) = \text{Exposure } (N) \cdot \text{Accident rate } (p)$$

$$P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}$$



Common measures of exposure and their weaknesses

- AADT
- Entering vehicles$_{major}$, entering vehicles$_{minor}$
- Annual kilometres of driving

- Often mixes very different types of road users and may not include all of them (pedestrians and cyclists are rarely counted)
- Averages over conditions representing different levels of risk
- Relationship to the number of accidents is often highly non-linear
- Different composite measures of exposure can be developed

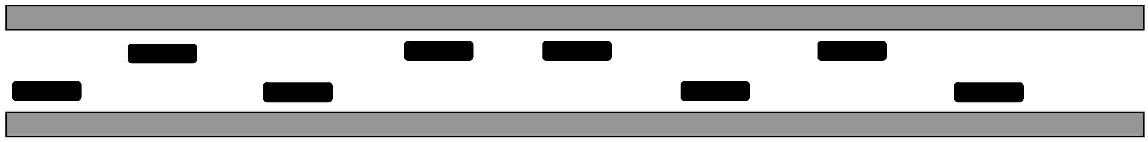
An example given by Hauer (2004)

	Leading	Lagging
Accidents	15	11
Left turn volume	2500	2500
Straight ahead volume	15000	10500
Measure of exposure	Accident rates	Accident rates
Left turn	4.11	3.01
Left + straight ahead	0.58	0.58
Left * straight ahead	0.40	0.42
	Which is safer?	Leading or lagging?

Event-based measures of exposure derived from summary measures

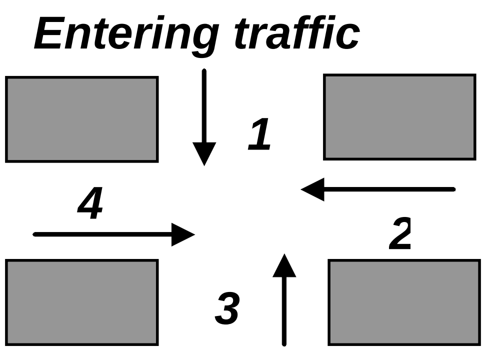
- Exposure is defined as events that create opportunities for accidents to occur
- Events should be clearly defined and should be countable
- Elementary events:
  - Encounters (vehicles passing each other in opposite directions)
  - Simultaneous arrivals from potentially conflicting directions
  - Lane changes
  - Braking or stopping
- Can be clearly associated with specific types of accident

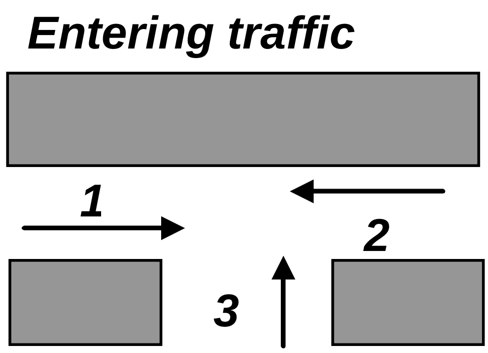
Encounters



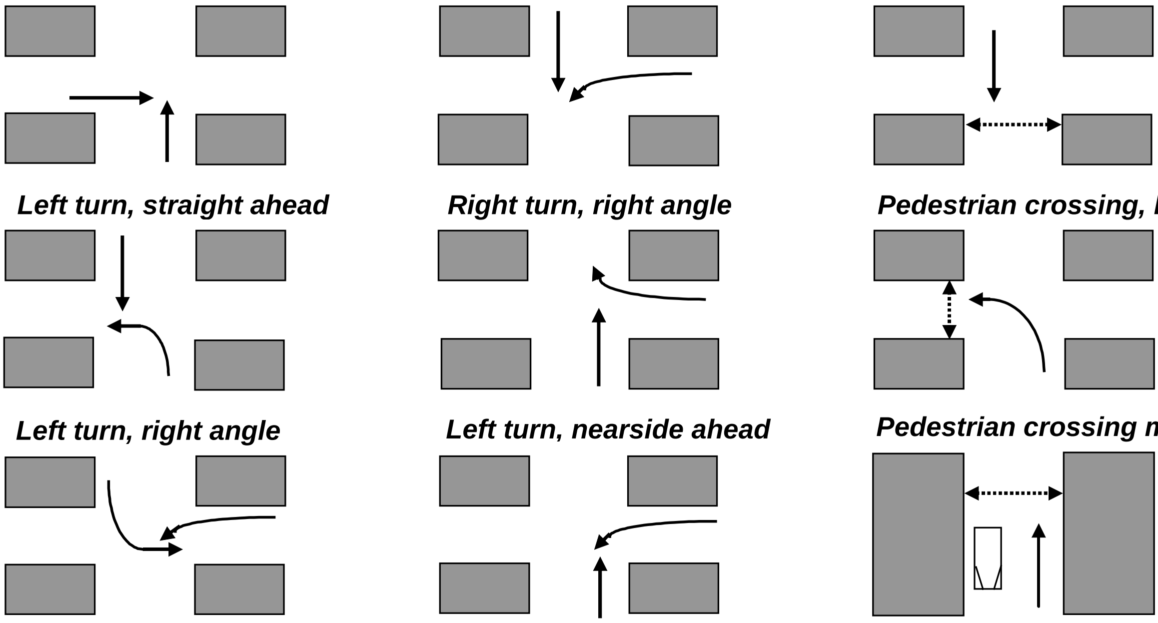
$$\text{Number of encounters} = \left(\frac{\text{Number of vehicles in both directions per unit of time}}{2}\right)^2$$

Simultaneous arrivals





A sample of potential conflicts



Simultaneous arrivals at junctions

Approach 1	Approach 2	Approach 3	Probability
116.7/3600	116.7/3600	116.7/3600	λ = 0.0324
0	0	0	0.9073
1	0	0	
0	1	0	0.0897
0	0	1	
1	1	0	
1	0	1	0.0030
0	1	1	
1	1	1	0.0000

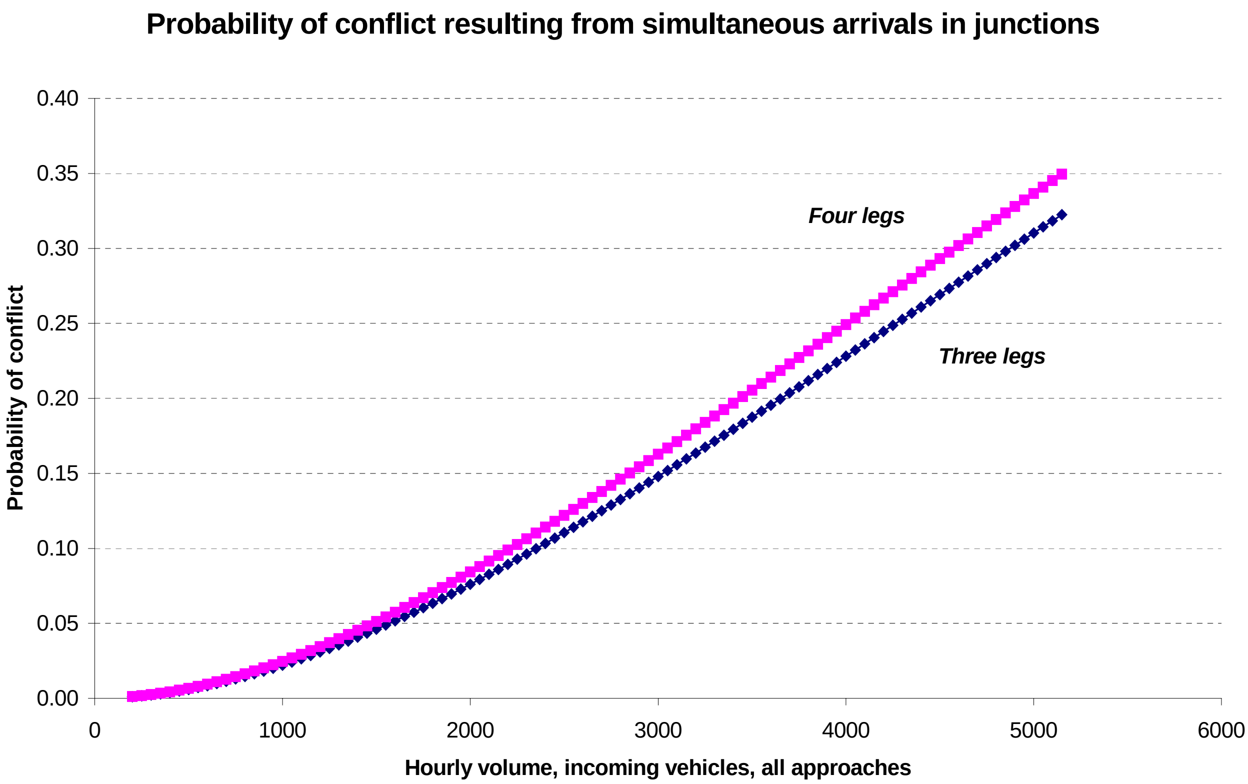
A closed-form solution

Mean number of arrivals per approach per second = λ = hourly volume/3600

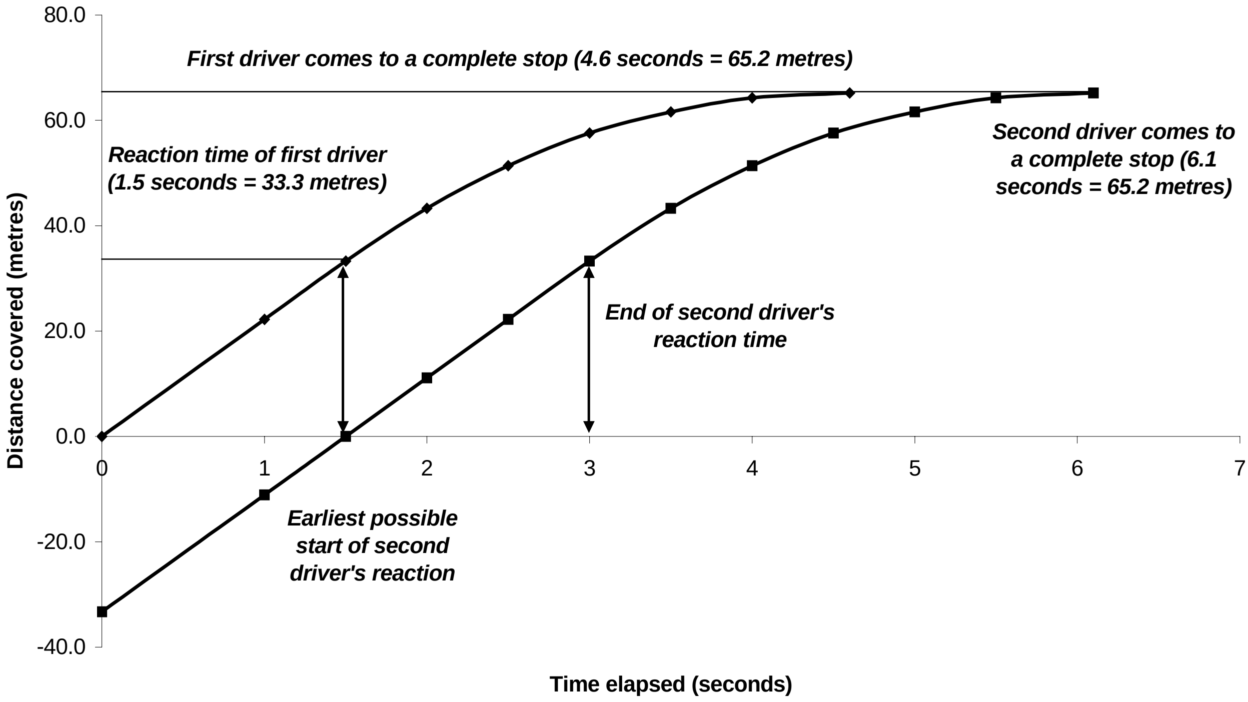
Probability of zero arrivals per second per approach = $e^{-\lambda} = M$

Probability of conflict in three leg junction = $2 \cdot M^3 - 3M^2 + 1$

Probability of conflict in four leg junction = $3 \cdot M^4 - 4M^3 + 1$



Braking to a complete stop from an initial speed of 80 km/h



Modelling probability of rear-end conflict

- Arrivals within reaction time are decisive
- Expected number of arrivals within reaction time = hourly lane volume/3600/1.5
- Estimate the probability of no car arriving ? no conflict
- Estimate the probability that 1 car will arrive ? no conflict
- Estimate the probability that a second car will arrive within the next 1.5 seconds ? potential conflict
- Estimate the probability that two more cars will arrive within the next 2 · 1.5 seconds, three more cars within the next 3 · 1.5 seconds, etc … up to 8 cars

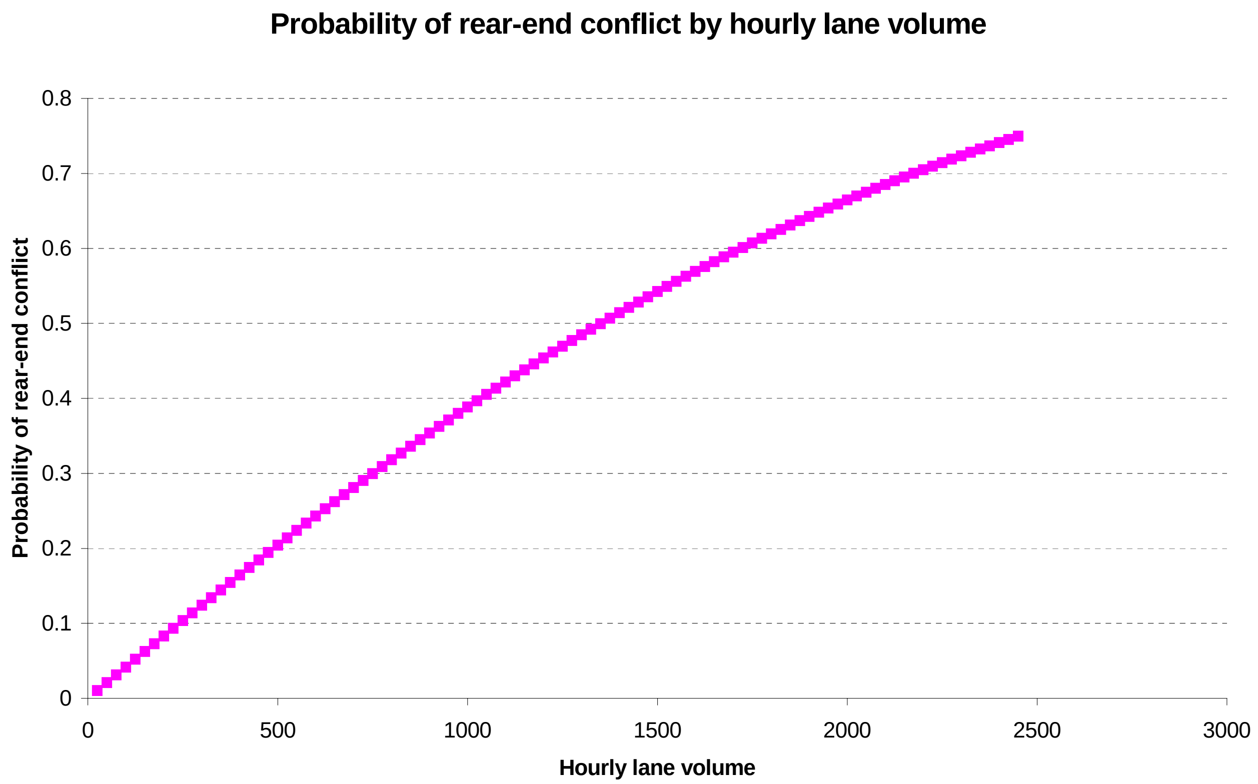
Closed-form solution for rear-end

Expected number of cars arriving within 1.5 second period = hourly lane volume/3600/1.5 = λ

Probability of 0 cars arriving within 1.5 seconds = $e^{-\lambda} = M$

Probability of 1 or more cars arriving within 1.5 seconds = $M^0 \cdot (1 - M^1)$

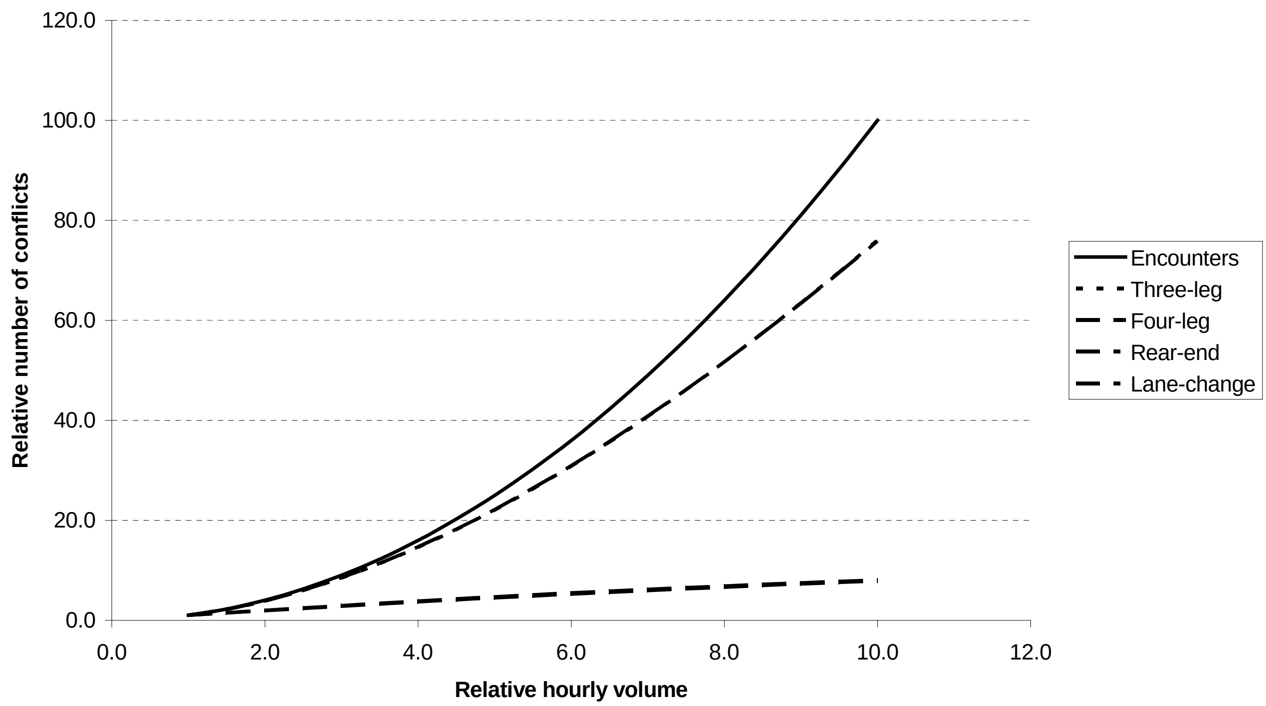
Probability of rear-end conflict = $1 - \lambda \cdot (M^2 + M^3 + M^4) - \frac{\lambda^2}{2} \cdot (M^3 + M^4) \frac{\lambda^3}{6} \cdot M^4 - M^4$



Lane changes

- If, at a random point in time, a decision is made to change lanes, the probability of a conflict depends on the probability that another car arrives in the adjacent lane within a short time interval
- A short time interval will be set equal to 1 second
- What is the probability of a car arriving (or occupying) the adjacent lane?
- Probability is modelled according to Poisson arrivals
- Probability of conflict = $1 - e^{-\lambda}$

Relative change in number of conflicts by relative hourly volume



Shape of relationship between exposure and accident rate

- If exposure is properly measured, the relationship should always be negative
- The larger the number of events, the lower the rate of accidents per event
- Why?
- Exposure represents opportunities for learning; the more road users are exposed, the better they learn how to identify and control hazards associated with specific types of events and variants of these

Implications

- The objective of defining exposure metrics that can isolate a constant risk of accidents across all levels of exposure is not realistic
- The main reason for this is that all forms of exposure, no matter how they are defined operationally, involve humans who do not want to become involved in accidents and whose skills in avoiding accident involvement tend to improve with increasing practice
- This applies irrespective of whether we study driver exposure or measures of exposure related to elements of road infrastructure