PARAMETERS DETERMINING ROUTE CHOICE IN PEDESTRIAN NETWORKS

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ABSTRACT

The focus of this paper is the modelling of a decision process that takes place on the tactical level of a pedestrian's trip. The tactical level is defined in delimitation to the superior strategic level and subordinated operational level with respect to trip purpose and spatial relations. Whereas on the strategic level the purpose, the origin and destination, the choices for traffic mode and time of departure are being set before the trip starts, on the tactical level, decisions are being made for the actual route or diversions within the pedestrian's network during the trip. At the tactical level, the decision making process can be modelled by the minimisation problem of walking costs in a network that takes into account both the network related quality and individual related factors. In the paper, the concept of a pedestrian quality attribute and its evaluation by physical assessable factors are introduced. In order to enable the application of the model, it is assumed that experiences in principle are gained by the walking pedestrian from prior knowledge of the walking network. This is the essential precondition for the decision making process that is based on a conventional route search algorithm. Instead the routing decisions by pedestrians are drawn in a mental process during walking that is characterised as non-formalistic. The purpose of the formalistic approach described in the text is to help understand the influencing factors and mechanisms of the decision making process in the analysis of pedestrian quality needs.

INTRODUCTION

For the analysis and modelling of decision processes, a spatial model of a walkable network is introduced. This walkable network model is utilised for the valuation of network related pedestrian quality as the basis for tactical routing decisions including individual human factors. The pedestrian quality is attributed to network elements according to a concept of virtual distances as the result of the valuation process. The decisions for route choices are being made under assumptions of utility maximisation of the individual pedestrian. In that way the utility maximisation follows the principles of finding the shortest paths in a network attributed by virtual distances. Thus, the model of a walkable network is introduced in the following sections as well as the quantification of pedestrian quality as a network attribute is described by means of an example.

MODEL OF A WALKABLE NETWORK

DEFINITION OF NODES AND LINKS

A walkable network is regarded as a directed graph consisting of nodes and links characterised by topological connectivity, attributes and geographic features, as illustrated in Figure 1. A node of the walkable network is defined at each point where a pedestrian can make a decision for his travel direction, typically intersections of sidewalks along roads, partings of ways and also places, squares or similar. On the tactical level also a place of some spatial extent, like a market place, is regarded as one node of the network. A link of the walkable network refers to a traffic connection from node to node that can be traversed by pedestrians usually in both directions. The direction of the link often refers to the direction of its construction, i.e. the digitising process or a predefined direction like chainage of the road documentation. Knowledge about the link direction is important for the reference of direction dependent attributes of network elements, for example differing pedestrian qualities on each travel direction.

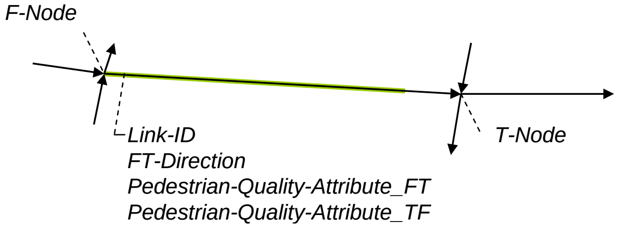


Figure 1 Elements of a walkable network model with pedestrian quality attributes *PQA*

Typical examples of links in a walkable network besides sidewalks are also greenways in parks or footpaths in gardens, way-leaves through buildings, passage-ways, pedestrian bridges or underpasses that are important traffic connections. It is evident that urban pedestrian networks are generally more complex than the underlying road network that can be referenced by a digital street map. Topological connectivity as a precondition for routing refers to the model of the walkable network in order to enable a search for a path through the network. In theory, for a pedestrian it is usually possible to use any connecting link from any given network node for routing unless its use is restricted by fences or barriers which is modelled by a table of forbidden turns.

SOURCES FOR MODELS OF PEDESTRIAN NETWORKS

A number of sources can be utilised to create a model of the walkable network for a selected area in a city. Commercial providers of digital road maps make their products available in various formats for further processing in a Geographic Information System (GIS) or spatial databases. Digital road maps contain both the geometry and useful attributes for each link in the network. The licences for use must be purchased and may be restricted to dedicated application cases. The *OpenStreetMap* project is a free editable map which is continuously extended and supplemented by a community of users using GPS tracking data. Geometric data of line objects representing streets and paths can be exported into XML-format for further processing without the need of licensing. Additional sources of geometries and link attributes are road databases, street information databases and pavement management systems operated and administered by municipal authorities. Regardless of the various sources of information, a unified database structure is necessary to represent the walkable network of the concerned area. General requirements for this structure include geometries, bidirectional link attributes and topological connectivity.

WALKING ZONES AND FREELY NAVIGABLE AREAS

The node/link model for a pedestrian network can not be applied within areas of considerable extent like market places, pedestrian zones or town squares. As the movement of individuals in these areas are relatively unrestricted with respect to direction, location of direction change, interference with other pedestrians, attractive and repulsive forces etc., the modelling approach for this typical operational walking behaviour requires a higher degree of freedom in two dimensional motion. A solution to this theoretical problem was proposed by Helbing and Molnár, (1995) through the introduction of the Social force model. It was suggested that the movement of pedestrians can be described as if they would be subject to 'social forces'. These forces are not directly exerted by the pedestrians' personal environment, but they are a measure for the internal motivations of the individuals.

The equations of this model include a term describing the acceleration towards the desired velocity of motion, terms reflecting a pedestrians' intention to keep a certain distance to other pedestrians and borders and finally a term modelling the attractive effects. This microscopic model was implemented in the simulation software *PedWalk*, a multi-agent rule based behavioural simulation package included as part of *BotWorld*, see Bruse (2002). The problems of pedestrian behaviour like motion in freely navigable areas, evasive manoeuvres, group forming and lane formation reside at the operational level and will not be discussed here. In connection with the evaluation of individual pedestrian related factors concerning route choice on the tactical level, only the social interaction is considered.

VALUATION OF NETWORK RELATED QUALITY

The evaluation of a walkable network includes the quantification of quality factors according to the method explained in this chapter. The physical measurable data of street planning and construction such as sidewalk width may be collected through GIS and road infrastructure databases operated by local authorities. The majority of factors need to be assessed in the field through audits supported by evaluation checklists.

INTRODUCTION OF THE PEDESTRIAN QUALITY ATTRIBUTE

The valuation of the network elements with respect to pedestrian quality aims to provide a methodology to quantify physical prevalent factors by either measurement or estimation in the field or through planning documentation.

For valuation of pedestrian quality the term quality attribute q_P is introduced. The quality attribute is the weighed sum of quality sub-attributes in the categories safety q_S, accessibility q_{Ac}, attractiveness q_{At} and comfort q_C.

The Pedestrian Quality Attribute PQA is given by

$$PQA = q_P = a_S q_S + a_{Ac} q_{Ac} + a_{At} q_{At} + a_C q_C \qquad (4\text{-}1)$$

with

$$0 < a_S, a_{Ac}, a_{At}, a_C < 1 \quad \text{and} \quad -1 < q_P, q_S, q_{Ac}, q_{At}, q_C < 1 \qquad (4\text{-}2)$$

where the domain of q_P ranges from poor (-1) to excellent (1). Each of the quality sub-attributes are weighed by the corresponding weighing coefficients a_S, a_{Ac}, a_{At} and a_C. For each quality category a defined number of quality factors contribute to the total quality that is to be assigned for each link in both travel directions. For example the quality category q_S is given by

$$q_S = \frac{1}{n}\sum_{i=1}^{n} q_S[i] \tag{4-3}$$

with quality factors $q_{S[i]}$. In this chapter only one factor per category will be examined in detail for each category. However the method is suited for any amount of data that is available in the field to be of use for the evaluation process.

PHYSICAL ASSESSABLE FACTORS

Amongst the great variety of physical factors influencing and determining pedestrian quality, the most important have been selected from the author's point of view to demonstrate the proposed method of evaluation of network elements. For each of the categories, safety, accessibility, attractiveness and comfort are analysed in more detail. Their suitability for physical quantification and the degree of impact within an aggregate quality attribute is documented.

Quality category - Safety

In this category two quality factors are examined further, safety of crossing facilities and motor traffic volumes and speed. As the importance of this category is assumed to be greater than for the categories of comfort and attractiveness it will be rated higher by 10 percent overvaluation. This amounts to a weighing coefficient of a_S =0,275.

Quality factor - Safe crossing facilities Signalised intersections and traffic light controlled crossing facilities for pedestrians (pelican crossings) are important contributions for improved safety in the traffic network. Therefore the existence of safe crossing facilities along a road segment constitutes the most important quality factor in this category. The minimum distance of mid-block crosswalks, pedestrian refuge islands or pelican crossings can be defined as 300 m in between. If this distance is exceeded on a link, the risk is raised that school children will not obey the safe crossing. Therefore any additional distance to the next pelican crossing or crosswalk reduces the safety and pedestrian quality. Let d_{Cr} denote the distance between crossing facilities, then the quality factor $q_{S,Cr}$ can be determined with

$$q_{S,Cr} = f(d_{Cr}) \tag{4-4}$$

where the function $f(d_{Cr})$ is qualified by the graph shown in Figure 2.

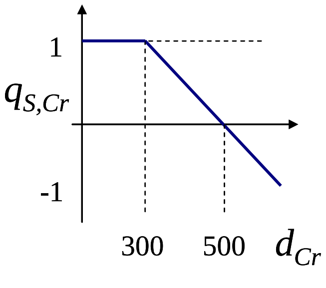
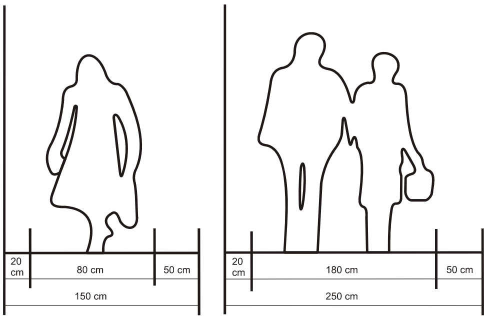


Figure 2 Relationship between distance of crossing facilities and quality factor $q_{S,Cr}$

Quality category - Accessibility

In this category two quality factors will be examined further, sufficient width of sidewalks and steepness of slopes. As the importance of this category is assumed to be greater than for the categories of comfort and attractiveness it will be rated higher by 10 percent overvaluation. This amounts to a weighing coefficient of a_{Ac} =0,275.

Quality factor - Sufficient width of sidewalks The most significant factor related to the criteria of accessibility is an adequate supply of sidewalk width depending on the demand of pedestrian traffic. For the planning and construction of sidewalks, technical guidelines exist that define the minimum requirements and necessary additions for safety clearance distances to buildings, roads, shop windows, trees, bicycle stands etc. Figure 3 illustrates the minimum case of necessary sidewalk width to provide the entry level of pedestrian quality.



Measure	**Distance**
Clearance to building	20 cm
Room for one person	80 cm
Room for two persons	180 cm
Clearance to road curb	50 cm
Total (for 2 persons)	250 cm

Figure 3 Minimum requirements for sidewalk width

In order to assess the factorial quality of width as part of the total quality, the existing width of a link must be measured. The quotient of existing width divided by planned minimum width for the link is called the width ratio w. The width ratio relates to the quality factor $q_{S,W}$ according to the linear relationship shown in Figure 4.

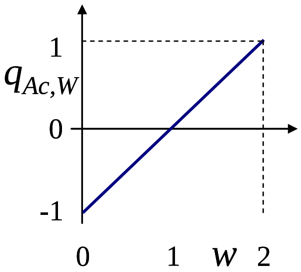


Figure 4 Relationship between width ratio and quality factor $q_{Ac,W}$

Quality category - Attractiveness

For the quality category of attractiveness, the factors maintenance of open space and lighting are considered. According to a 10% lower evaluation for this category the weighing coefficient is assigned to a_{At} =0,225.

Quality factor - Lighting The illuminance is a photometric measure of the luminous flux per unit area of light. Illuminance at the surface of a pavement is the normative technical measure for the perceived brightness. A spatially evenness of illuminance on the pavement is important for the recognition of obstacles, persons and vehicles. The SI unit for illuminance is lux (lx) or lumen per square metre (lm/m^2) and can be measured with a lux meter. For pedestrian facilities located along streets or in parks and residential neighbourhoods, the minimum requirements for illuminance are defined in national standards, as shown in Figure 5.

Facility - Area	**Illuminance in lx**
Urban pedestrian zone	5
Urban market places and squares	5
at temporarily high pedestrian traffic volume	10
Greenways in parks	1
Crosswalks	3
Stairways inside	100
outside	15
Arcades and passages	15
Underground crossing	50-100

Figure 5 Illuminance requirements for urban pedestrian facilities in Germany

For the assessment of the factorial quality of sufficient illuminance of a given network segment as part of the total quality, the present value must be measured at the surface of the pavement. The quotient of measured illuminance divided by required minimum illuminance for the examined link is called the illuminance ratio i. The illuminance ratio i relates to the quality factor $q_{At,L}$ according to the linear relationship presented in Figure 6.

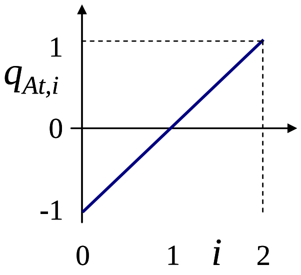


Figure 6 Relationship between illuminance ratio and quality factor $q_{At,L}$

Quality category - Comfort

The quality category of comfort includes two quality factors, noise level mainly of motorised traffic and vegetation and cast of shadow. The importance of this category is assumed to be lower than for the categories of safety and accessibility. Therefore the corresponding weighing coefficient is set to a_C =0,225.

Quality factor - Noise level The noise level induced from individual motorised traffic correlates with speed and volume of cars and heavy trucks. Higher noise levels along sidewalks of arterial roads will most likely lower the comfort level and quality of the referring pedestrian facility. In the case of available alternative routes the chances are high that a routing decision will follow a more comfortable way as long as all other factors and the distances are equal. Noise as unwanted sound is measured physically in decibels as sound pressure level (A), where the A denotes a filter that attempts to adjust sound measurements to correspond to loudness as perceived by the average human. Some examples of sound pressure are given in the table below.

Source of sound	Sound pressure level in dB (A)
Leaves rustling	10
Calm room	30
Talking person	50
Car at 10 m distance	60-80
Arterial road at 10 m distance	80-90
Jack hammer at 1 m distance	100
Pain threshold	130

Figure 7 Examples of sound pressure

Sound pressure level can be directly measured by an instrument such as a sound level meter over a defined period of time. In direct measurement the value as logarithmic measure in dB is displayed at the meter. For evaluation of the measured average values, a functional relationship is given in Figure 8. In the example displayed, a measured sound level of 70 dB(A) will result in a reduction of quality factor $q_{C,N}$ by 50 percent. The non-linear relationship is justified under the circumstances that raising sound pressure levels that are likely to occur along urban roads will lead to a substantial aggravation of pedestrian quality.

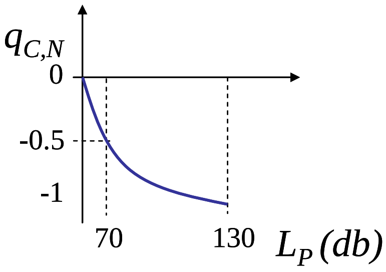


Figure 8 Noise level quality factor $q_{C,N}$ as a function of average sound pressure

Evaluation example

For the evaluation of pedestrian quality, a sidewalk along an urban road section close to a tourist location in the city of Magdeburg, Germany serves as an example, see Figure 9. The street is named Breiter Weg, the evaluated section extends from *The Cathedral* to *The Green Citadel (Hundertwasserhaus)* with a total length of about 320 m. In reference to the NavTeQ^{TM} street map this road section is identified by ID=77281317. The evaluation is carried out for the westerly situated sidewalk. The reference direction is determined by the driving direction for right hand traffic which is indicated by the magenta arrow in Figure 9. However, the corresponding NavTeQ^{TM} link (blue arrow) is oriented in the opposite direction. Thus, the value of quality attribute PQA to be determined must be assigned to the field PQA_TF of a database table that refers to the digital street map. An evaluation for the pedestrian quality of the westerly located sidewalk consists of the determination of quality factors of the equation

$$PQA\_TF = q_P = a_S q_S + a_{Ac} q_{Ac} + a_{At} q_{At} + a_C q_C \qquad (4\text{-}5)$$

Figure 9 Evaluated sidewalk on west side of Breiter Weg, Magdeburg, Germany

As a result of quantification of physically measured values, each quality factor is determined using the functional relationships given above. Two quality factors are averaged for each quality category, weighed and summarised as the value of the total pedestrian quality for the given direction. The table in Figure 10 summarises the calculations for the evaluated example. The total PQA results in the positive value of 0,428 which is to be interpreted as an explicitly attractive momentum that will lead to a preference of this section in the mental route planning.

Quality category	**Quality factor**	**Physical value**	**Quality symbol**	**Value**	**Mean**	**Weigh factor**	**PQA**
Safety	Crossing facilites	285 m	$q_{S,Cr}$	1	0,85	0,275	0,234
	Traffic volumes	1000 @ 30 km/h	$q_{S,V}$	0,7			
Accessibility	Width of sidewalk	10 m	$q_{Ac,W}$	1	0,5	0,275	0,138
	Steepness of slopes	0%	$q_{Ac,Sl}$	0			
Attractiveness	Maintenance of space	A	$q_{At,M}$	1	1	0,225	0,225
	Lighting	10 lx	$q_{At,L}$	1			
Comfort	Noise level	70 dB(A)	$q_{C,N}$	-0,5	-0,75	0,225	-0,169
	Vegetation/Shadow	< 10%	$q_{C,Sh}$	-1			
						Total	0,428

Figure 10 Calculation of the Pedestrian Quality Attribute PQA for the given direction

SOFT FACTORS AND SOCIAL FORCES

Besides physical assessable quality factors, also various soft factors or social forces can lead to either attracting or repelling pedestrians to parts of the network and influencing their routing decisions. These factors have in common that prior knowledge must be available to the individual pedestrian about their kind and location. If these soft factors exist temporarily, an influence on a routing decision can only be assumed if it is visible to the individual at the point of decision. Examples for attractions are possibilities of social interaction like groups of persons, street artists, street markets and temporary exhibitions or street festivals. Examples for repelling factors are socially insecure places like known crime spots and areas known for loitering and beggary, as well as alcohol and drug abuse. The valuation of soft factors, as an increase or decrease of the quality attribute PQA, can be realised by estimating the social force factor aSF for each concerned network element.

The domain of a_{SF} is defined as

$$-1 < a_{SF} < 1 \tag{4-6}$$

valued from repulsion (-1) to attraction (1). The social force factor is added to the evaluated link related pedestrian quality attribute PQA. As such, the social force factor serves as an additive measure for the further increase or decrease of the virtual distance between nodes of the network.

$$WA = \frac{1}{2}(PQA + a_{SF}) \tag{4-7}$$

The resulting attribute is denoted as walkability attribute WA and measures the cost for travelling the network paths. Decisions for route choice are drawn during the routing process that determines the shortest virtual path.

WALKABILITY ATTRIBUTE AS MEASURE OF VIRTUAL DISTANCE

As mentioned in the preceding section, the pedestrian quality attribute is determined by evaluating the walkable network by means of quality factors. Hard quality factors are directly measurable whereas soft quality factors are assessable by estimations. As a result of the evaluation process, there are two values for PQA obtained, each one for both directions of the link. The quality attribute, if the impact of soft factors is also included, will be denoted as walkability attribute *WA*. The walkability attribute is utilised to define a measure for the virtual distance that is essential for a routing decision that takes into account the link quality and social factors. The basic idea is to include the pedestrian quality into the routing decision with the concept that poor quality, i.e. a low attribute value will increase the "felt" distance for the pedestrian. An increased "felt" distance, further denoted as virtual distance, will lower the chances of a link to be chosen at a decision point if an alternative way exists, that features at least the same virtual distance. Otherwise, this virtual distance decreases at a positive quality assessment of the link that is more attractive. In the process of utility maximisation which is presumed as a basis for the routing decision, always the shortest virtual distance will be chosen by the pedestrian.

VALUATION OF PEDESTRIAN RELATED FACTORS

Apart from quality related factors there are important human factors that will have a strong impact on routing decisions on the tactical level. The trip purpose, personal fitness as well as time constraints will have a significant influence on route choice. However, on the tactical level these influences are regarded to be constant, since it is not expected that these factors will change during a trip. Hence the individual factors are considered as additional input quantities for the utility maximisation process of route choice that will influence the decisions evenly over the entire network. The human factors are therefore subject to considerations at the strategic level, but the kind and dimension of impact on the tactical level will be examined in the following section.

THE INDIVIDUAL TRIP PURPOSE

The purpose of a trip has a major influence on the way of taking individual on-trip routing decisions. Within the model discussed so far, pedestrian quality is related to the network only. Therefore, the individual factor of the trip purpose determines to which extent the quality of the network will be taken into account when the routing decision is being made. Two scenarios from the perspective of the commuter (the impatient traveller) and the tourist (the patient traveller) act as examples to consider the main differences with respect to route choice.

Commuter trip to work place with time constraints - The impatient traveller

For a commuter trip under the time constraint of a scheduled arrival time it is assumed that an individual pedestrian will take the most effective route from home to workplace, or in a multimodal trip from public traffic stop to destination. This case can also be characterised by an impatient traveller in a hurry to an appointment. In general, the chosen route in this case is the shortest path between the starting point and the end point throughout the pedestrian network. As such, the physical distance of network links is the exclusive basis for route choice of the commuter. Hence, it is used the geographical length of network links as routing attribute RA.

$$RA := length(network\_links) \tag{4-8}$$

The exact route could be found through a routing algorithm using the physical distance as a measure to weigh the travelling costs to be minimised. In practice, the pedestrian will carry out the route planning in a similar way, although this is accomplished without calculating the exact distances. By using a map, the human being is intuitively able to determine the shortest path between two given points in a network through alignment of the planned path as close as possible to the straight connection.

Leisure walking without time constraints - The patient traveller

For the trip purpose of a leisure walk or stroll, or any trip done by a patient traveller without any strict time constraints, the quality aspect is assumed to be most important. This case can be modelled by applying the walkability attribute WA as the exclusive measure to weigh the travelling costs for minimisation by the route search.

$$RA := -WA(network\_links) \tag{4-9}$$

A pedestrian route in a quality weighed network is optimal if the travelling costs over the network links are minimal. Since positive values of WA represent an increased quality they are inverted for use as a routing attribute that is to be minimised. The travelling costs for each link are determined using the valuation procedure for the network related pedestrian quality described above. Therefore a route search algorithm using the walkability attribute would find a different route than the shortest physical distance between origin and destination. The route choice of an individual pedestrian would rely on the same preconditions that are valid for a routing algorithm under the circumstances of quality preference.

PEDESTRIAN'S UTILITY MAXIMISATION

In the preceding sections it was analysed under which circumstances pedestrians choose their route in a walkable network from origin to an intended destination under the assumption of previously acquired knowledge about the network. The common assumption that pedestrians normally choose the shortest available route was extended by the introduction of the Pedestrian Quality Attribute PQA as a measure of virtual distance in the attributed network that incorporates the pedestrian quality as part of the walking cost.

Since route search algorithms depend on an attributed network to determine the optimal route under the given preferences, the appropriate selection or combination of attributes is essential for the intended use case to be modelled. As discussed above, it is necessary to use the spatial distance to attribute the network in order to model the case of a commuter trip. The virtual distance as a measure of walking costs was introduced to take into account the pedestrian quality and human factors into the concept of routing. This virtual distance is related to a link as a network element which can enlarge or reduce the "felt" physical distance depending on the decrease or increase of quality. Concerning quantification of the virtual distance it is required that the attribute value must be positive, since any route search algorithms solve the shortest path problem only for networks with nonnegative path costs.

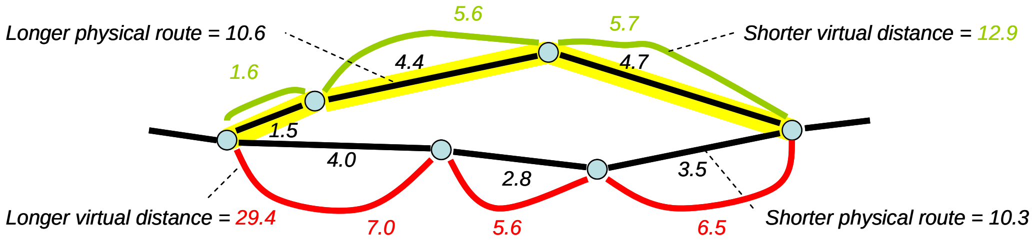


Figure 11 Route choice based on virtual distances

The example of Figure 11 illustrates the case of using the virtual distance instead of the physical distances. The shorter physical route between two alternative paths has a length of 10.3. The longer physical route has a length of 10.6. By including quality aspects the (red) virtual distance of the short route is 29.4, whereas the longer route has a (green) virtual distance of 12.9. Assume for the pedestrian the maximisation of utility under no time constraint, then the quality is more important than the route shortness. Therefore the shorter virtual attributes will be used for the route search that will finally yield to the (yellow) route.

MINIMISATION OF COSTS THROUGH ROUTE SEARCH

In order to be able to use the discussed model of analysing parameters of route choice, it is necessary to emphasise that an individual pedestrian needs to acquire prior knowledge about the quality of the network along the route towards his destination and about possible existing alternatives as a precondition for making decisions on route choice. In the presented considerations it is assumed that a pedestrian is informed about the network quality from experiences of earlier walks. Under these circumstances the route search at the tactical level can be carried out by a graph search algorithm that solves the shortest path problem in the cost attributed network. The application Dijkstra's (1959) algorithm and other extended or derived algorithms such as A* or Bellman (1958) – Ford (1962) deliver the solution according to specified premises. The necessary preconditions for the application of the route search model are defined as follows:

- Origin and Destination as start and end nodes of the considered pedestrian network are known and set.
- Alternatives of more that one route from origin to destination exist in the network.

- The network quality is evaluated and attributed to any affected link in both directions.
- The individual factors as trip purpose and user abilities are defined.

After the definition of preconditions, the routing decisions are found by the following steps:

1. Given the attributed network, the virtual distance is calculated as a function of the Pedestrian Quality Attribute PQA for each link of the network.
2. Given the virtual distance of each network link and the spatial distances, the total walking costs are assigned whether using the virtual distance or the spatial distance depending on individual preferences such as time constraints and abilities.
3. Given the total walking costs, the route is determined as a result of the utility maximisation by minimising the costs over all paths through any path finding algorithm such as the Dijkstra algorithm.

The routing decisions result from the calculated route and are related to the given attributed network under defined preconditions and assumptions.

CONCLUSION

Routing decisions of pedestrians are strongly related to the network quality and individual factors. For the tactical level, that is on the trip during walking, the decision making process of route choice can be modelled by minimising the problem of walking costs that take into account both the network related quality and individual related factors. That is assumed under the precondition of acquired prior knowledge and assessment of the walking network by the pedestrian. Individual factors, such as time constraints and physical abilities are incorporated in the model as they influence the weigh of attributes used in the process of maximising the personal utility of the human individual.

ACKNOWLEDGEMENTS

The authors would like to express their acknowledgement to the European Science Foundation (ESF) for granting an international project within the research framework of the European Cooperation in Science and Technology (COST) under grant number PQN-358. The results of which contributed to this paper.

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