

White Paper
SURROGATE MEASURES OF SAFETY

ANB20(3) Subcommittee on Surrogate Measures of Safety
ANB20 Committee on Safety Data Evaluation and Analysis

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April 28, 2009

MOTIVATION AND PURPOSE OF THIS PAPER

Crash frequency and severity are direct measures of road safety. Therefore, road safety analysis has traditionally been undertaken using crash data. However, there are well-recognized availability and quality problems associated with crash data. Crash data are not always sufficient due to:

1. Small sample sizes leading to inconclusive results, and
2. The lack of details to improve our understanding of crash failure mechanism and especially the driver crash avoidance behavior.

Furthermore, the use of crash records for safety analysis is a reactive approach: a significant number of crashes need to be recorded before an action can be taken. This also reduces the ability to examine the safety effects of a recently implemented safety countermeasure. Because of these issues, road safety analysis can benefit from reliable analysis methods that utilize observable non-crash traffic events and other surrogate data instead of the accumulation of crashes.

The current impressive progress in sensing technologies and in statistical methodologies makes possible developing valid and practical surrogate-based methods of estimating and modeling safety. The FHWA Workshop on New Directions in Modeling Crash Data held in November, 2008, has identified three most promising research directions. Research on surrogate measures of safety was among them.

The Highway Safety Manual (HSM) is the anticipated product of a current major effort to develop a leading reference for safety analysts and engineers. To serve as a major resource, the HSM should include not only standard but also emerging methods of safety evaluation. For example, the Highway Capacity Manual in recent years has recognized micro-simulation as an alternative approach to highway capacity analysis and has embraced this new methodology as an approach complementary to more traditional analytic-based approaches. Similarly, the new HSM may consider surrogate measures as a viable and complementary approach to safety evaluation that may be used separately or jointly with crash data, provided these methods are deemed viable, accurate, and defensible.

The Subcommittee on Surrogate Measures of Safety is sponsored by the TRB Committee on Safety Data Evaluation and Analysis – ANB20. The goal of this subcommittee is to examine the suitability and use of surrogate measures of safety to cope with the lack of available crash data. This goal will be pursued by the Subcommittee through the development of a synthesis of existing knowledge and available methods, organizing special conference sessions, and encouraging implementation of existing and emerging methods through guidelines and workshops.

This white paper is aimed to: (1) Clarify the concepts, terms, and definitions of surrogate measures of safety, (2) Summarize past research on surrogate measures and point out their weaknesses, (3) Indicate new ideas in this area, and (4) Identify research directions leading to successful development and use of surrogate measures of safety.

CONCEPT OF SURROGACY

Many surrogate measures of safety have been proposed and used, but the concept remains somewhat vague. In the medical sciences surrogate measures are used relatively often. The commonly accepted clinically meaningful outcome of a medical treatment is quality of life after the treatment. Because this outcome cannot be measured during the treatment, surrogate outcomes are necessary to evaluate the treatment beforehand. A large number of research publications on various surrogate measures (markers, outcomes) can be found via the on-line database search engine *Entrez* at <http://www.ncbi.nlm.nih.gov/Database/index.html>. For example, the level of lipoprotein cholesterol in patients with coronary heart disease has long been considered a suitable surrogate for the likelihood of the disease-inflicted death.

To serve as an acceptable surrogate, a surrogate measure has to meet two conditions:

1. It is correlated with the clinically meaningful outcome, and
2. It fully captures the effect of the treatment.

In the area of road safety the engineering meaningful outcome is the reduction or elimination of target crashes. Reduction of the frequency of events necessary for these crashes to happen is an appealing surrogate outcome that meets the first condition set forth in the medical sciences. Thus, the frequency and other characteristics of such surrogate events may be considered promising surrogate measures of safety. Figure 1 presents how surrogate measures and safety are related. The necessity of a surrogate event for a corresponding crash to happen establishes a causal relationship represented by the two events depicted by the horizontal arrow in Figure 1. Safety may also be affected by factors that are external to the surrogate measures. These factors are depicted by the vertical arrow. The fewer and weaker are these additional factors, the more powerful is the surrogate measure. For example, traffic events reminiscent of crashes are believed to share many factors with crashes.

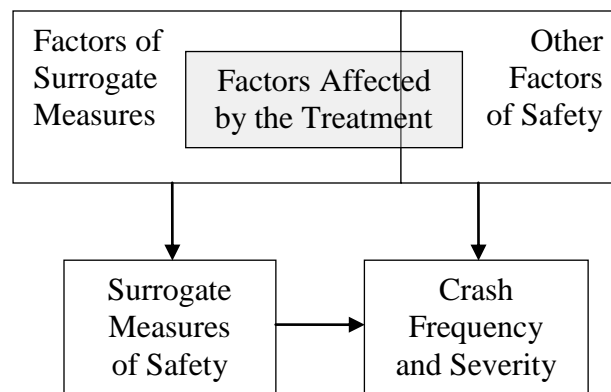


Figure 1 Relationship between surrogate measures and safety

The second condition of a good surrogate measure established in the medical sciences is that it captures the treatment effect. In our application area, it means that a surrogate event must be affected by the engineering treatment if the treatment indeed affects safety. Obviously, the more powerful the surrogate measure, the easier it is to meet this condition.

To be useful for transportation safety applications, a surrogate measure should satisfy two conditions:

1. A surrogate measure should be based on an observable non-crash event that is physically related in a predictable and reliable way to crashes, and
2. There exists a practical method for converting the non-crash events into a corresponding crash frequency and/or severity.

The first condition emphasizes the crucial aspects of crash surrogacy that enable meeting the second conditions: development of a method of converting the surrogate outcomes into the meaningful outcome – frequency and severity of crashes. The main difference between medical sciences and transportation engineering is that the latter needs a practical way of converting the surrogate measures to numerical crash equivalency, while the former accepts correlation between the surrogate measure and the clinically meaningful outcome.

According to the above discussion, traffic volume meets the first condition of a good surrogate measure. Vehicles present on the road are necessary for crashes to happen. This measure has, however, a limited use. It seldom meets the second condition because most of the safety treatments do not affect traffic volume. One of a few exceptions is diverting traffic from dangerous to less dangerous routes.

Speed is proposed and used by some authors as a safety surrogate measure. Indeed, it should be included as an important component of a surrogate event definition. Its use, however, as a standalone surrogate measure may be difficult due to the complexity of the speed-safety relationship. Converting the change in speed to the change in crash frequencies may be difficult.

In each of these examples the main analytical concerns surround the predictability and reliability of the surrogate measure with safety outcomes in addition to the physical relationship with safety.

PAST RESEARCH

Multiple authors have attempted to identify and/or apply surrogate measures of safety to overcome the frequent lack of sufficient crash data. Some examples include: traffic conflicts (Chin et al., 1992; Chin and Quek, 1997; Glauz and Migletz, 1980; Parker and Zegeer, 1989), critical events, e.g., aggressive lane merging, speeding, and running on red (Kloeden et al., 1997; Porter et al., 1999); acceleration noise (Shoarian-Sattari and Powell, 1987); post-encroachment time (Allen et al., 1978); and time-integrated time-to-collision (Minderhoud and Bovy, 2001).

These traffic conflict indicators and others are discussed in more details in (van der Horst, 1990, Gettman and Head, 2003, Archer, 2004). Other proposed measures are volume, speed, delay, accepted gaps, headways, shock-waves, and deceleration-to-safety-time (FHWA, 1981).

The most prevalent measures considered by highway safety engineers are traffic conflicts and their frequency. The most mature versions of traffic conflict technique (TCT) use time to collision and conflicting speed (Hydén, 1987; Hydén, 1996; Svensson, 1998; Svensson and Hydén, 2006). A traffic conflict best addressed the condition of a good surrogate measure, namely, common factors shared with safety if it is severe (close proximity of crash). The core assumption of the current TCT is that conflicts can be defined in a way that imposes a constant coefficient between crashes and conflicts (Hauer, 1982) or at least limits its variance (Hauer and Garder, 1986).

In a study by Sayed and Zein (1999), regression analyses was used to develop predictive models that relate the number of traffic conflicts to traffic volumes and accidents from 92 intersections. Both conflicts and accidents were assumed to follow a Poisson distribution. A statistically significant relationship was found between accidents and conflicts with an R^2 in the range of 0.70 - 0.77 at signalized junctions but not at unsignalized intersections.

Zhang *et al.* (2006) defined the concept of crash “opportunities” as a measurement exposure that not only accounts for trials but also the opportunity for crashes to occur. This definition may be considered a further generalization of traffic conflicts.

Mixed success has been reported for the estimation and validation of coefficients converting traffic conflict frequencies to crash frequencies, a process that is furthermore expensive and troublesome (see one of the latest attempts by Tiwari, et al., 1998). Nevertheless, traffic conflicts in the present version offer benefits that warrant the use of this technique as a method complementary to crash-based analysis.

A traditional approach utilized in the vast area of research on human factors is to identify behavioral characteristics of drivers and other road users that are undisputedly related to crash risk and severity of crashes. Although this approach is not quantitative, it allows concluding whether or not the road improvement affects safety and in what direction. Recent applications of various surrogate measures to evaluate the safety of modern road solutions can be found in Great Britain and Sweden (Malkhamah et al., 2005; Lu, 2006; Johansson and Leden, 2007). These surrogate measures follow the concept of surrogacy defined for the medical sciences and does not fully meet the conditions discussed earlier for the road safety because there is no method of converting the surrogate values to the corresponding crash frequency and severity.

PROMISING APPROACHES AND CHALLENGES

The Extreme Value Method

Quite recently, a new idea of estimating the frequency of crashes based measured crash proximity is described in Songchitruksa and Tarko (2004) and it is based on the Extreme Value

Theory introduced in a formal way in (Haan and Ferreira, 2006) and statistical analysis of extreme values described in (Reiss and Thomas, 2007). The proposed approach, called here the Extreme Value Method (EVM), considers interactions between vehicles as risky events (called here, surrogate events) and estimates the risk of crash conditional on such an event based on observed separations between interacting vehicles. The EVM represents three considerable advantages over the traffic conflict technique:

1. The EVM abandons the assumption of a fixed coefficient converting the surrogate event frequency into the crash frequency,
2. The risk of crash given the surrogate event is estimated for any conditions based on the observed variability of crash proximity without using crash data,
3. The crash proximity measure precisely defines the surrogate event.

The crash frequency is a product of measured frequency of risky events $F(H)$ and the estimated conditional likelihood of crash $P(C|H)$:

$$F(C) = F(H) \cdot P(C | H)$$

where

$F(C)$ = estimated frequency of crashes at the road location,

$F(H)$ = observed frequency of surrogate events H at the road location,

$P(C|H)$ = risk of crash given surrogate event H estimated with the extreme value model developed for the road location.

The method's basic concepts are explained in detail and preliminarily evaluated elsewhere (Songchitruksa and Tarko, 2004; Songchitruksa, 2004). The method in its simplified version has been applied to estimate the frequency of right-angle collisions at signalized intersections (Songchitruksa and Tarko, 2006b). The surrogate event selected in these studies involved two vehicles passing the crossing point of their paths shortly one after another (post-encroachment time shorter than six seconds).

Although the EVM method has been conceived for fast measuring of the crash frequency, it can be extended to build crash frequency models much more efficiently than based on crash data. The crash frequency could be predicted given the estimated relationships between safety and road and other conditions X :

$$F(C | X) = F(H | X) \cdot P(C | H, X)$$

where

$F(C|X)$ = frequency of crashes under a set of conditions $X=(X_1, X_2, \dots X_n)$,

$F(H|X)$ = count model of surrogate event H frequency,

$P(C|H,X)$ = extreme value model of risk of crash given surrogate event H and conditions X .

Research on the EVM is needed to overcome the weaknesses of the current TCT. This research should include the following research components leading to development, evaluation, and implementation:

1. Identification of the crash proximity measures that fit the EVM concept; different types of interactions (surrogate events) and corresponding crashes require different measures of crash proximity,
2. Identification of the conditions and EV models for efficient and unbiased estimation of the frequency of crashes,
3. Testing and evaluation of the EVM with measurements collected in driving simulators and/or collected on the road with instrumented vehicles and road sensors,
4. Development of guidelines and manuals to help implement the EVM.

Counterfactual Approach

Another related area of research is development of effective and accurate methods of measuring vehicle interactions, such as distance separation between vehicles and time separation between events. One promising, but still experimental, method for extracting crash surrogates from such data is by applying Pearl's (2000) theory of probabilistic causal models (Davis, Hourdos and Xiong, 2008).

In the literature, it is possible to find two related, but different, approaches to defining crash surrogate events. One is the definition of conflict as put forward by the International Calibration Study of Traffic Conflict Techniques (ICSTCT):

A traffic conflict is an observable situation in which two or more road users approach each other in space and time to such an extent that there is a risk of collision if their movements remain unchanged.

The other is the definition of near crash used in the 100-Car Study:

A near crash is any circumstance that requires a rapid evasive maneuver by the subject vehicle, or any other vehicle, pedestrian, cyclist, or animal to avoid a crash. A rapid, evasive maneuver is defined as a steering, braking, accelerating or any combination of control inputs that approaches the limit of the vehicle capabilities.

Both of these imply a counterfactual definition of the surrogate event, where had an evasive action not been performed a crash would have occurred. The 100-Car Study definition places an additional condition however, that the magnitude of the evasive action is some sense extreme. This second condition helps exclude situations such as when two vehicles successively brake to a stop at an intersection with one stopping just behind the other. A small decrease in the following driver's deceleration would lead to a rear-end crash, but if the decelerations were typical of what drivers use when stopping this does not seem sufficient to consider the event a near-crash.

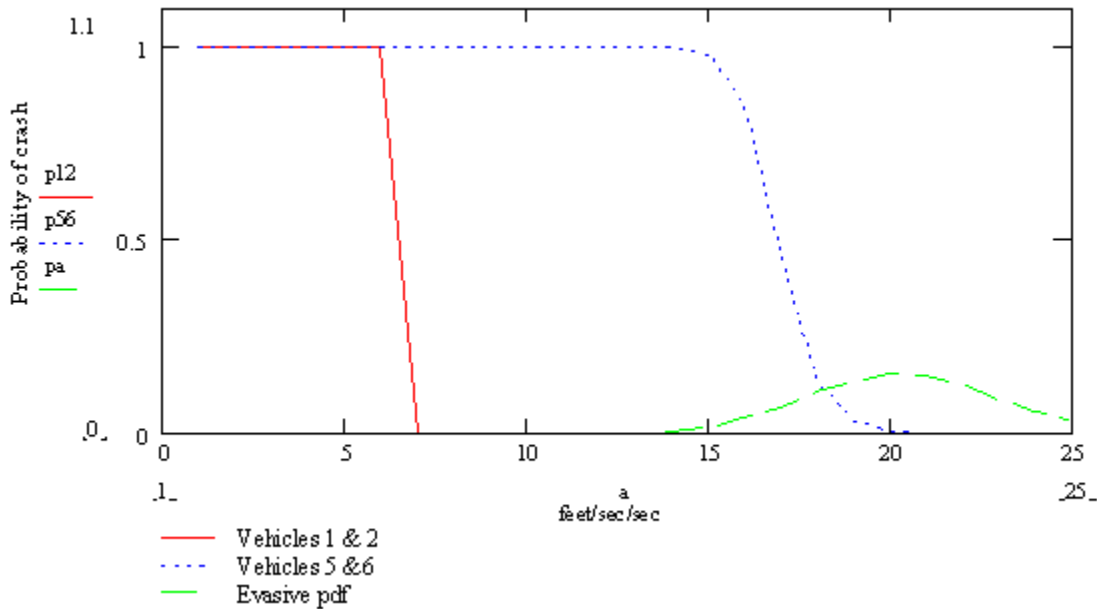


Figure 2. Crash probability versus following vehicle deceleration, and probability density function for emergency decelerations.

Figure 2 plots probability of crash versus counterfactual deceleration of the following vehicle for two pairs of vehicles observed to stop on US I-94, during congested conditions. The initial vehicle speeds, the drivers' following distances, reaction times and deceleration rates were estimated from video-extracted trajectory data, and the residual uncertainty in these estimates makes crash occurrence probabilistic. For the first pair of vehicles, decelerations on the part of the following driver below about 6 ft/sec² are needed to make a crash highly probable, while for the second pair this occurs for decelerations below about 16 ft/sec². Intuitively, the second event seems more like a near crash than the first, and this intuition can be quantified if one can identify a distribution of evasive actions characteristic of crash/near-crash conditions. Figure 2 also shows such a distribution, developed from statistics reported in the study by Fambro et al (1997) of braking under emergency conditions. Integrating the crash probability vs deceleration curve with respect to this distribution gives a probability an event could have been a crash, in this case essentially zero for the first pair of vehicles but about 0.14 for the second. Computing these probabilities for each of a set of events and then summing these probabilities gives an expected number of crashes in a set of near-crash events.

A Probabilistic Framework for Automated Road Safety Analysis

Considering the shortcomings of traditional traffic conflict indicators such as the time to collision, Saunier and Sayed (2008), presented a comprehensive probabilistic framework for automated road safety analysis. They provided computation definitions of the probability of collisions for road users involved in an interaction and calculated aggregated measures over time.

This framework is suitable for an automated system that can extract individual detailed road user movement information, typically from video data. Such a system requires a high level understanding of the scene and is traditionally composed of two levels of modules:

1. A video processing module for road user detection and tracking,
2. Interpretation modules for interaction analysis and traffic conflict detection.

For road safety applications, the approach relies on the building of two databases:

1. A trajectory database, where the results of the video processing module are stored,
2. An interaction database, where all interactions between road users within a given distance are considered. For each interaction, various indicators, including collision probability and other severity indicators, are automatically computed.

Identifying traffic conflicts and measuring other traffic parameters becomes the problem of mining these databases.

The collision probability for a given interaction between two road users can be computed at a given instant by summing the collision probability over all possible motions that lead to a collision, given the road users' states. This requires the ability to generate for each road user at any instant a distribution over its possible future positions given its previous positions. A possible future motion, i.e. a temporal series of predicted positions, defines an extrapolation hypothesis. The collision probability computation is approximated by a discrete sum when taking into account a finite number of the most probable extrapolation hypotheses. [More details is given in Saunier and Sayed, 2008]

Such a system can provide detailed severity measures, exposure estimates and a method to detect and study traffic conflicts. The system was demonstrated using real traffic data, including some traffic conflict instances, with the results showing the feasibility and usefulness of the system. Further research is needed to investigate and validate the relationship of collision probability to safety.

Challenges

Research should focus on the testing and validation of safety surrogates making use of new analytical and measurement technologies. Complexities in the selection, testing, and validation of appropriate safety surrogates need to be subjected to the research rigors applied in other areas of safety research. These measures need to be subjected to the peer-review and evaluation process and corroborated through multiple studies and researchers. Only through a new emphasis on the area of safety surrogates and their validation by a body of researchers will their use be justified by practicing engineers.

Looking forward, work is needed to address the scaling problem of converting surrogate frequencies to on road crash frequencies. This will undoubtedly require synergistic research between human factors researchers and on-road researchers, with focus on the analytical challenges of scaling and the necessary validation of such approaches.

Another significant challenge is the selection of surrogates that are reliable with respect to their physical relationship with crashes. Consider the use of speeding as a surrogate. It is poorly understood as a surrogate for safety. Many agencies and research reports have cited speeding as a major contributing factor (i.e. surrogate) in crashes. The problem, however, is that the vast majority of data on speeding are conditioned on a crash occurring, and provide the conditional probability of speeding. If speeding were to be observed on a roadway, in contrast, it would be unconditional. Since we do not understand the unconditional relationship between speeding and crashes, speeding may not be a reliable predictor of a crash. For example, it may be true that for some drivers that very rarely speed their crash probability is quite high (e.g. older drivers), while other drivers that speed quite often are rarely involved in crashes. If this is true, even for a portion of the population, the reliability of speeding as a surrogate is in question.

Consider as another example excessive braking maneuvers. One might presume that observing these at a location (e.g. intersection) may serve as a reliable predictor of crashes. However, some drivers that apply brakes aggressively are avoiding crashes, while others that fail to apply brakes are involved in crashes.

These two simple examples illustrate in a simple and perhaps naïve way the potential complexities involved in selecting reliable safety surrogates. At the very least these examples suggest that the relationship between surrogates and safety may be complex and involve explanatory variables that change the nature of the relationship. The examples also hint at the dire need to conduct extensive validation of selected surrogate measures.

CLOSURE

Research on surrogate measures of safety will benefit both safety engineering and safety science. Traffic crashes are pervasive road system failures, yet our understanding of the failure mechanism is poor, which reduces the accuracy of road safety diagnosis and the estimation of countermeasure effectiveness. Safety engineering desperately requires a breakthrough in safety evaluation, and safety surrogates may serve as one of those breakthroughs. Modern highway transportation systems, such as Active Safety Systems and Vehicle-Infrastructure Integration, once implemented, will dramatically change safety on our roads. Reliable and efficient safety evaluation tools are a necessity for providing assessment and feedback. The traditional evaluation method based on crash analysis will not be able to deliver timely safety estimates to match the safety-affecting progress in vehicles and in intelligent infrastructure.

Improvement in the theory of road safety associated with a better understanding of the link between road safety, driver behavior, and dynamic traffic interactions may reveal surrogate measures of safety observed directly on the road and used to estimate the crash frequency and severity of crashes. It is quite likely that the research on surrogate measures itself will advance the collective understanding of complex safety-related traffic interactions and will promote progress in the theory of road safety.

Technological and methodological developments allow optimism about our ability to measure safety on roads and in vehicles without requiring crash data. This ability will materialize through spurring interest among transportation researchers toward research on surrogate measures of safety, encouraging sponsors to fund such research, and promoting alternative methods of safety evaluation among road administration and transportation practitioners.

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